Full quantum theory for magnon transport in two-sublattice magnetic insulators and magnon junctions

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Outline

1. Background

   - Early theories for magnon transport

   - Magnon junction

2. Our works

3. Results and discussions

4. Conclusion and Outlook

Theory for magnon transport – LLG equation

Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\frac{\gamma}{\mu_s(1+\alpha^2)} \mathbf{S}_i \times [\mathbf{H}_i + \alpha(\mathbf{S}_i \times \mathbf{H}_i)]$$

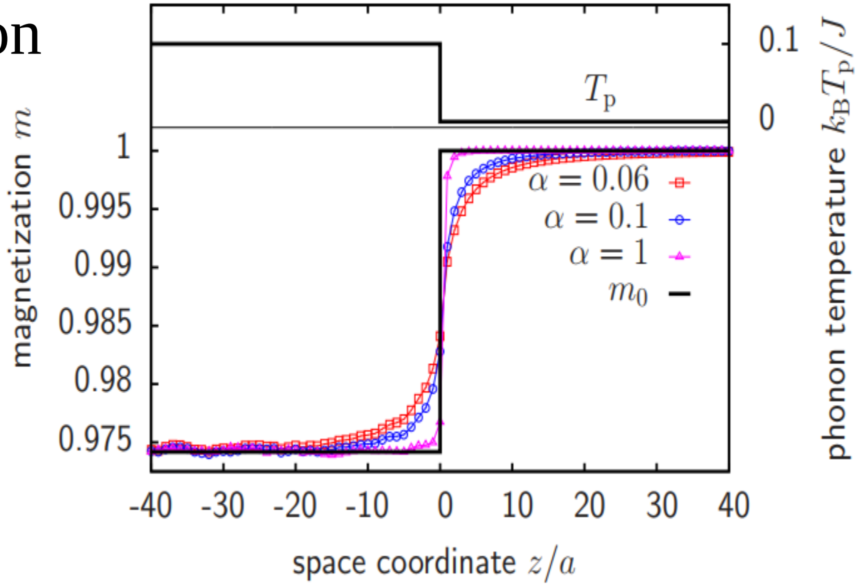
Where the effective field

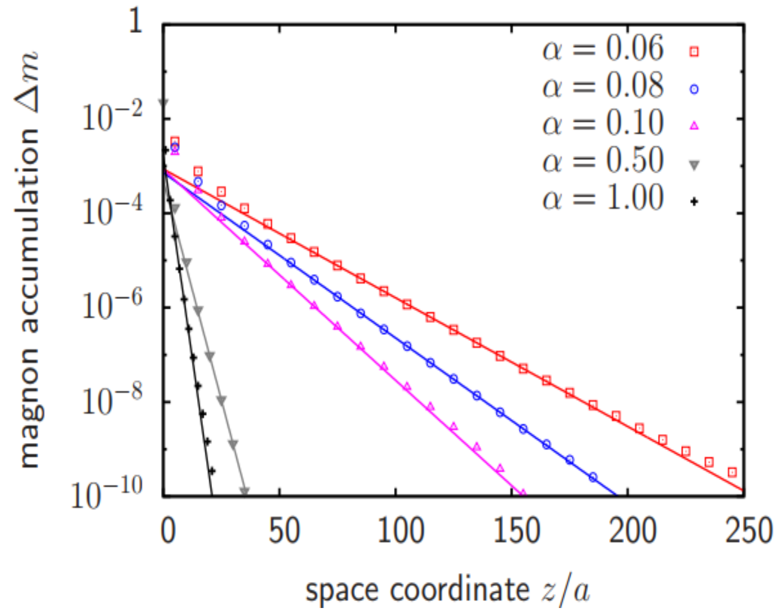
$$\mathbf{H}_i = -\frac{\partial \mathcal{H}}{\partial \mathbf{S}_i} + \zeta_i(t),$$

$$\mathcal{H} = -\frac{J}{2}\sum_{\langle i,j\rangle} \mathbf{S}_i \mathbf{S}_j - d_z \sum_i S_{i,z}^2$$

$$\langle \zeta(t) \rangle = 0,$$

$$\left\langle \zeta_\eta^i(0) \zeta_\theta^j(t) \right\rangle = \frac{2 k_B T_p \alpha \mu_s}{\gamma} \delta_{ij} \delta_{\eta\theta} \delta(t)$$





Ritzmann, et al, PRB, 89, 024409 (2014)
Ritzmann, et al, PRB, 95, 054411 (2017)

Theory for magnon transport – Magnon Schrodinger equation

LLG equation :

$$\frac{\partial \mathbf{S}_i}{\partial t} = -\frac{\gamma}{\mu_s(1+\alpha^2)} \mathbf{S}_i \times [\mathbf{H}_i + \alpha(\mathbf{S}_i \times \mathbf{H}_i)]$$

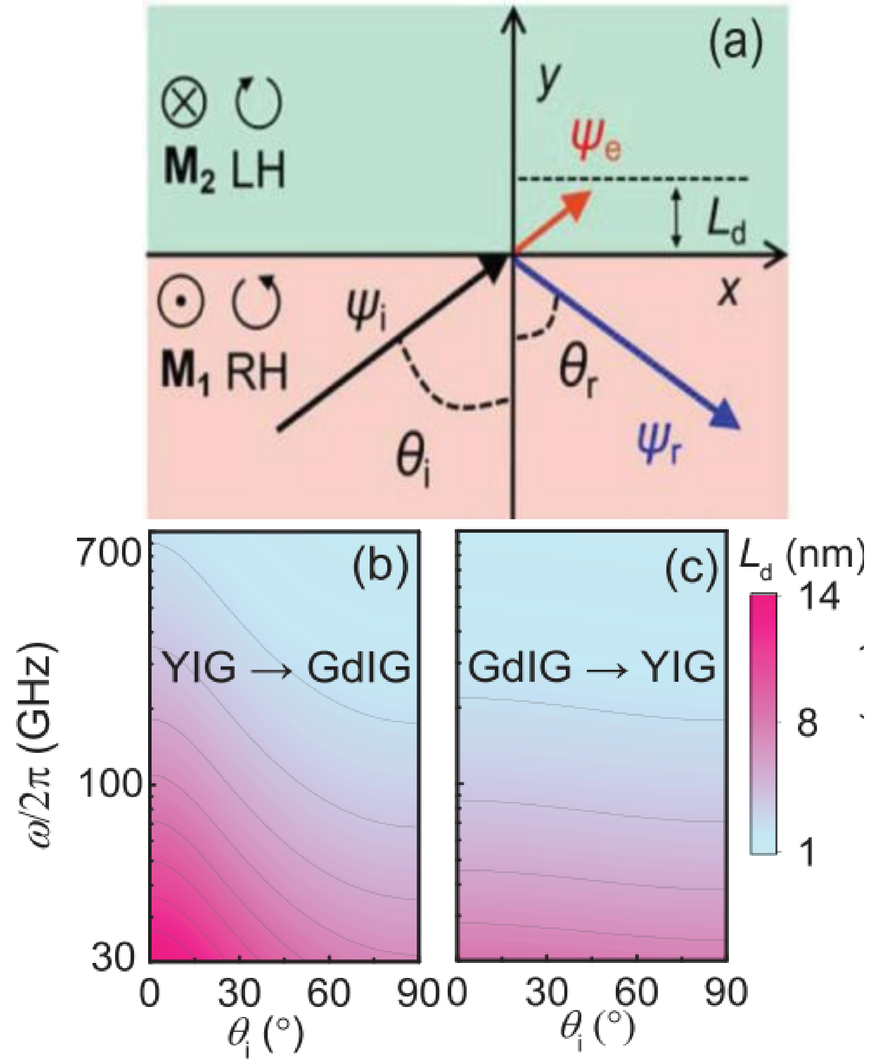
where $\mathbf{H}_i \;\; = \sigma_i \left(\frac{2K_i}{\mu_s} \hat{\boldsymbol{e}}_z + \frac{A_i}{\mu_s} \nabla^2 \mathbf{S}_i \right)$

Define $\psi_i = S_{x,i} - iS_{y,i}, \hat{p} = -i\hbar\nabla$

Magnon Schrodinger equation:

$$i\hbar \frac{\partial \psi_i}{\partial t} = \mathcal{H}_i \psi_i = \left[\frac{\hat{p}^2}{2m_i^*} + V_i \right] \psi_i$$

$$\Longrightarrow \psi_i \sim e^{i(\mathbf{k}\cdot\mathbf{r} - \omega_\eta t)}$$



Yan, et al, PRB, 104, L020413 (2021)

Theory for magnon transport – Magnon Green's function

Hamiltonian:

$$\widehat{H}_{tot} = \widehat{H}_{\mathrm{FM}} + \widehat{H}_{\mathrm{NM}} + \widehat{H}_{C}$$

Green's function:

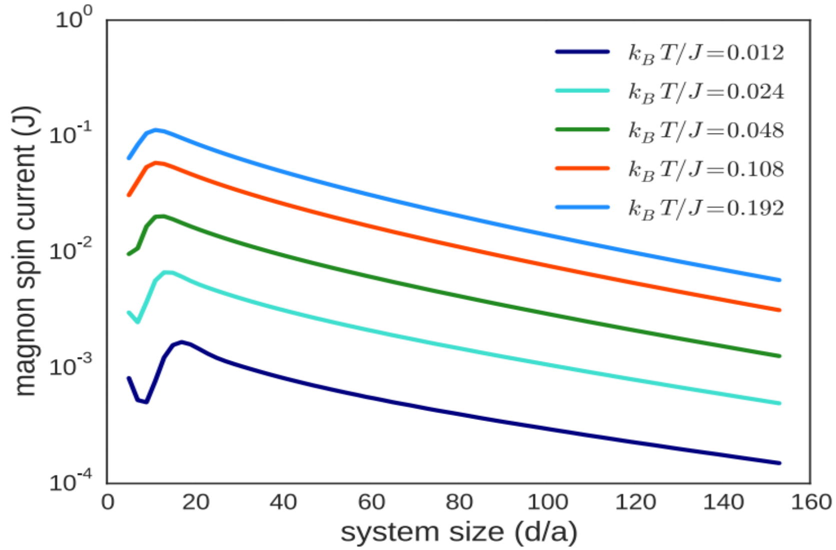
$$\left[\epsilon^{\pm} - h_{tot} - \hbar\Sigma^{R(A)}(\epsilon)\right]\mathcal{G}^{R(A)}(\epsilon) = 1$$

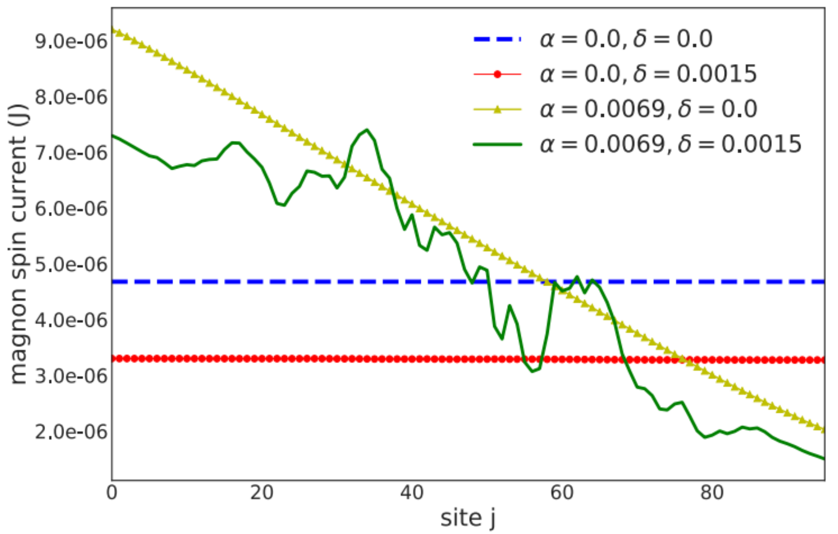
Density matric:

$$\rho_{j,j'} \equiv \left\langle \hat{b}^{\dagger}_{j'}(t)\hat{b}_{j}(t)\right\rangle$$
$$= \int \frac{d\epsilon}{(2\pi)}\left[\mathcal{G}^{(+)}(\epsilon)i\hbar\Sigma^{<}(\epsilon)\mathcal{G}^{(-)}(\epsilon)\right]_{j,j'}$$

Magnon current:

$$j_{m\,;\,j\,j'} = -i(h_{j,j'}\rho_{j',j} - c.c.)$$



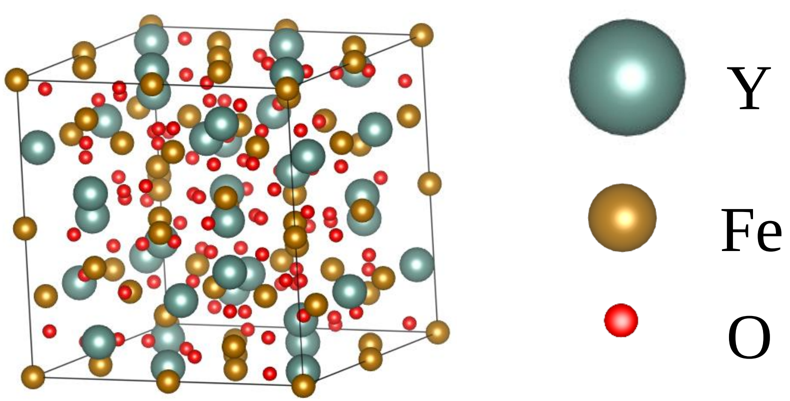


Zheng, et al, PRB, 96, 174422 (2017)
Sterk, et al, PRB, 104, 174404 (2021)

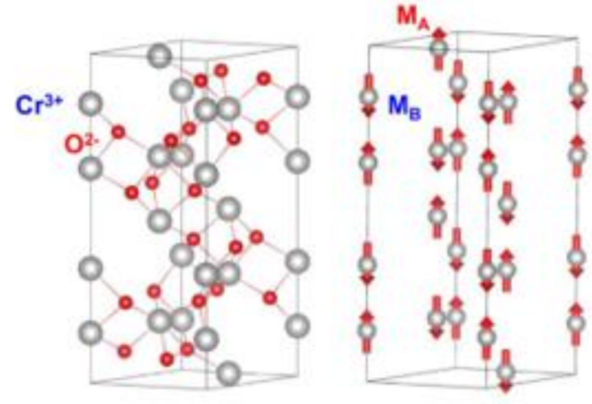
Theory for magnon transport

Theory for magnon transport	Advantages	Disadvantages
LLG equation	Easy to understand and calculate	Phenomenological theory, microscopic mechanism is not clear
Magnon Schrodinger equation	The research methods and conclusions of wave mechanics can be utilized.	Derived from LLG equation, phenomenological in nature
Magnon Green's function	Quantization theory; It is convenient to study the influence of the coupling on the magnon transport	All quantum theory for ferrimagnets and antiferromagnets is needed

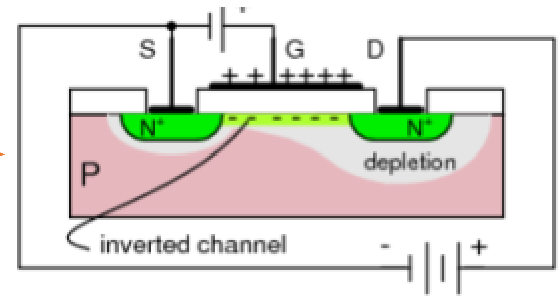
YIG (Yttrium Iron Garnet)

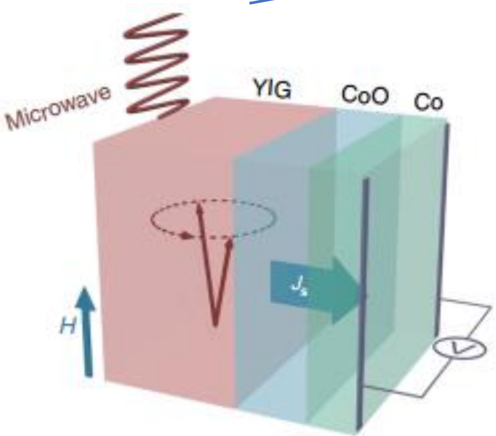


Cr_2O_3

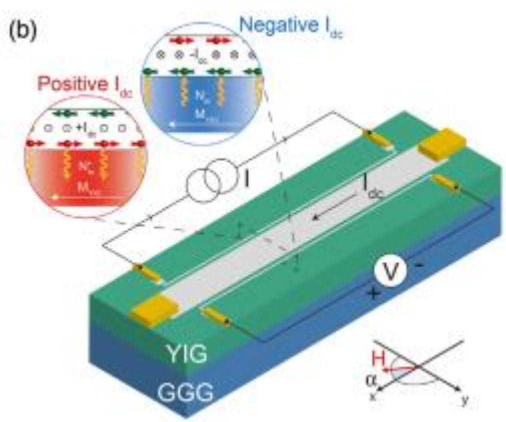


Magnon junction

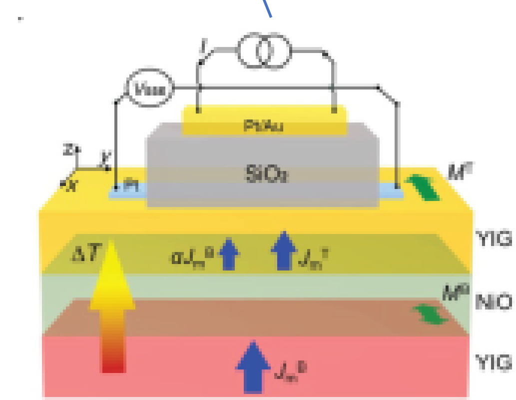




Cramer, et al, Nat. Commun, 9, 1089 (2018)



Cornelissen, et al, PRL, 120, 097702 (2018)



Guo, et al, PRB, 98, 134426 (2018)

Green's function formalism for magnon system

Hamiltonian of two-sublattice magnetic insulators:

$$\hat{H} = -J_{AB}\sum_{<i,m>} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_m - J_A \sum_{\ll i,j \gg} \hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j - J_B \sum_{\ll m,n \gg} \hat{\boldsymbol{S}}_m \cdot \hat{\boldsymbol{S}}_n - h_{\text{ext}}\left(\sum_i \mu_A \hat{S}_i^z + \sum_m \mu_B \hat{S}_m^z\right)$$

Using Holstein – Primakoff (H-P) transformation

$$\hat{S}_i^x = \frac{\sqrt{S_A}}{2}(\hat{a}_i + \hat{a}_i^+), \hat{S}_i^y = \frac{\sqrt{S_A}}{2i}(\hat{a}_i - \hat{a}_i^+), \hat{S}_i^z = S_A - \hat{a}_i^+\hat{a}_i;$$

$$\hat{S}_m^x = \frac{\sqrt{S_B}}{2}\left(\hat{b}_m^+ + \hat{b}_m\right), \hat{S}_m^y = \frac{\sqrt{S_B}}{2i}\left(\hat{b}_m^+ - \hat{b}_m\right), \hat{S}_m^z = S_B - \hat{b}_m^+\hat{b}_m$$

and Fourier transformation

$$\hat{a}_i^{(\dagger)} = \frac{1}{\sqrt{N}}\sum_k e^{(-)i\boldsymbol{k}\cdot\boldsymbol{R}_i}\hat{a}_k^{(\dagger)}, \hat{b}_m^{(\dagger)} = \frac{1}{\sqrt{N}}\sum_k e^{-(+)i\boldsymbol{k}\cdot\boldsymbol{R}_m}\hat{b}_k^{(\dagger)}$$

Green's function formalism for magnon system

$$
\begin{aligned}
\Longrightarrow \hat{H} = & \sum_k \left(-2J_A S_A \gamma_{k,nn} - J_{AB} S_B N_n + 2J_A S_A N_{nn} + h_{ext}\mu_A\right) \hat{a}_k^{\dagger} \hat{a}_k \\
& + \sum_k \left(-2J_B S_B \gamma_{k,nn} - J_{AB} S_A N_n + 2J_B S_B N_{nn} - h_{ext}\mu_B\right) \hat{b}_k^{\dagger} \hat{b}_k \\
& + \sum_k \left[-J_{AB}\sqrt{S_A S_B}\,\gamma_{k,n} \left(\boxed{\hat{a}_k \hat{b}_k} + \boxed{\hat{a}_k^{\dagger} \hat{b}_k^{\dagger}}\right)\right] \\
\equiv & \sum_k \left[A_k \hat{a}_k^{\dagger} \hat{a}_k + B_k \hat{b}_k^{\dagger} \hat{b}_k + C_k \left(\hat{a}_k \hat{b}_k + \hat{a}_k^{\dagger} \hat{b}_k^{\dagger}\right)\right]
\end{aligned}
$$

Using Bogoliubov transformation

$$
\begin{aligned}
\hat{a}_k = u_k \hat{\alpha}_k + v_k \hat{\beta}_k^{\dagger}, \ \hat{a}_k^{\dagger} = u_k \hat{\alpha}_k^{\dagger} + v_k \hat{\beta}_k, \\
\hat{b}_k = u_k \hat{\beta}_k + v_k \hat{\alpha}_k^{\dagger}, \ \hat{b}_k^{\dagger} = u_k \hat{\beta}_k^{\dagger} + v_k \hat{\alpha}_k
\end{aligned}
$$

Green’s function formalism for magnon system

$$\Longrightarrow \hat{H} = \sum_{k} [(A_k u_k^2 + B_k v_k^2 + 2C_k u_k v_k)\ \hat{a}_k^{\dagger} \hat{a}_k$$

$$+(A_k u_k v_k + B_k u_k v_k + C_k (u_k^2 + v_k^2)) \boxed{(\hat{\alpha}_k^{\dagger} \hat{\beta}_k^{\dagger} + \hat{\alpha}_k \hat{\beta}_k)}$$

$$+(A_k v_k^2 + B_k u_k^2 + 2C_k u_k v_k)\ \hat{\beta}_k^{\dagger} \hat{\beta}_k$$

$$\equiv \sum_k [A_k \hat{a}_k^{\dagger} \hat{a}_k + B_k \hat{b}_k^{\dagger} \hat{b}_k + C_k (\hat{a}_k \hat{b}_k + \hat{a}_k^{\dagger} \hat{b}_k^{\dagger})]$$

Take

$$u_k = -\sqrt{\frac{1}{2} + \frac{A_k + B_k}{2\sqrt{(A_k + B_k)^2 - 4C_k^2}}},\quad v_k = \sqrt{-\frac{1}{2} + \frac{A_k + B_k}{2\sqrt{(A_k + B_k)^2 - 4C_k^2}}}$$

Green's function formalism for magnon system

$$\Rightarrow \hat{H} = \sum_k [\frac{A_k - B_k}{2} + \frac{\sqrt{(A_k^2 + B_k^2) - 4C_k^2}}{2}] \hat{\alpha}_k^\dagger \hat{\alpha}_k + [\frac{-A_k + B_k}{2} + \frac{\sqrt{(A_k^2 + B_k^2) - 4C_k^2}}{2}] \hat{\beta}_k^\dagger \hat{\beta}_k$$

$$\equiv \sum_k [w_k^\alpha \hat{\alpha}_k^\dagger \hat{\alpha}_k + w_k^\beta \hat{\beta}_k^\dagger \hat{\beta}_k]$$

Methods	Functions
Holstein - Primakoff transformation	Express the Hamiltonian of the system using magnon annihilation and creation operators
Fourier transformation	Transfer representation from real space to reciprocal space
Bogoliubov transformation	Decouple two types of magnons.

Green's function formalism for magnon system

Steps to calculate α mode magnon current:

① Hamiltonian

$$\boxed{\hat{H}} = \sum_{<j,j'>} \left[A_0 \cdot \left(\delta_{j,j'+1} + \delta_{j,j'-1}\right) + A_1 \cdot \delta_{j,j'}\right] \hat{\alpha}_{j'}^{+} \hat{\alpha}_j$$

② Green's function

$$\boxed{\mathcal{G}^{R(A)}}(\epsilon) = [\epsilon - \boxed{h} - U(\epsilon) - \hbar\Sigma^{R(A)}(\epsilon)]^{-1}$$

③ Density matrix

$$\boxed{\rho_{j,j\prime}} = \int d\epsilon\, [\boxed{\mathcal{G}^R}(\epsilon) i\hbar\Sigma^{<}(\epsilon)\boxed{\mathcal{G}^A}(\epsilon)]_{j,j\prime}$$

④ Magnon current

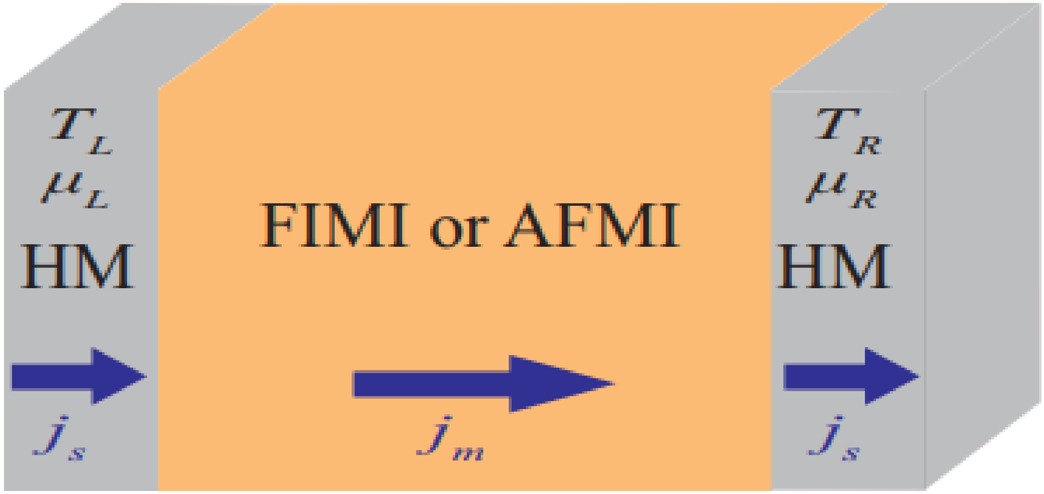
$$< j_{m,jj\prime} > = -i(h_{j,j\prime}\boxed{\rho_{j\prime,j}} - h_{j\prime,j}\boxed{\rho_{j,j\prime}})$$

$$j^{m}_{L(R),\alpha} = \int \frac{d\varepsilon}{2\pi}\left[N_B\left(\frac{\varepsilon - \mu_{L(R)}}{k_B T_{L(R)}}\right) - N_B\left(\frac{\varepsilon - \mu_R(L)}{k_B T_R(L)}\right)\right] T_{b,\alpha}(\varepsilon)$$

$$+ \int \frac{d\varepsilon}{2\pi}\left[N_B\left(\frac{\varepsilon - \mu_{LR)}}{k_B T_{L(R)}}\right) - N_B\left(\frac{\varepsilon - \mu_C}{k_B T_{AFMI}}\right)\right] T_{f,\alpha}(\varepsilon)$$

Results and discussions

Model:



- In this setup, the ferrimagnetic insulator (FIMI) or antiferromagnetic insulator (AFMI) is connected to two heavy metals (HMs) with temperatures T_R, T_L and spin chemical potentials μ_L, μ_R, respectively.
- The magnon current is driven by the difference of temperature or spin chemical potential between two HMs .

Results and discussions - AFMI

Parameters of (a, b):

$\mu_L = \mu_R = 0$

$k_B T_{AFMI} = 0.026$ eV

$T_L = 1.2\ T_{AFMI}$,

$T_R = 0.8\ T_{AFMI}$

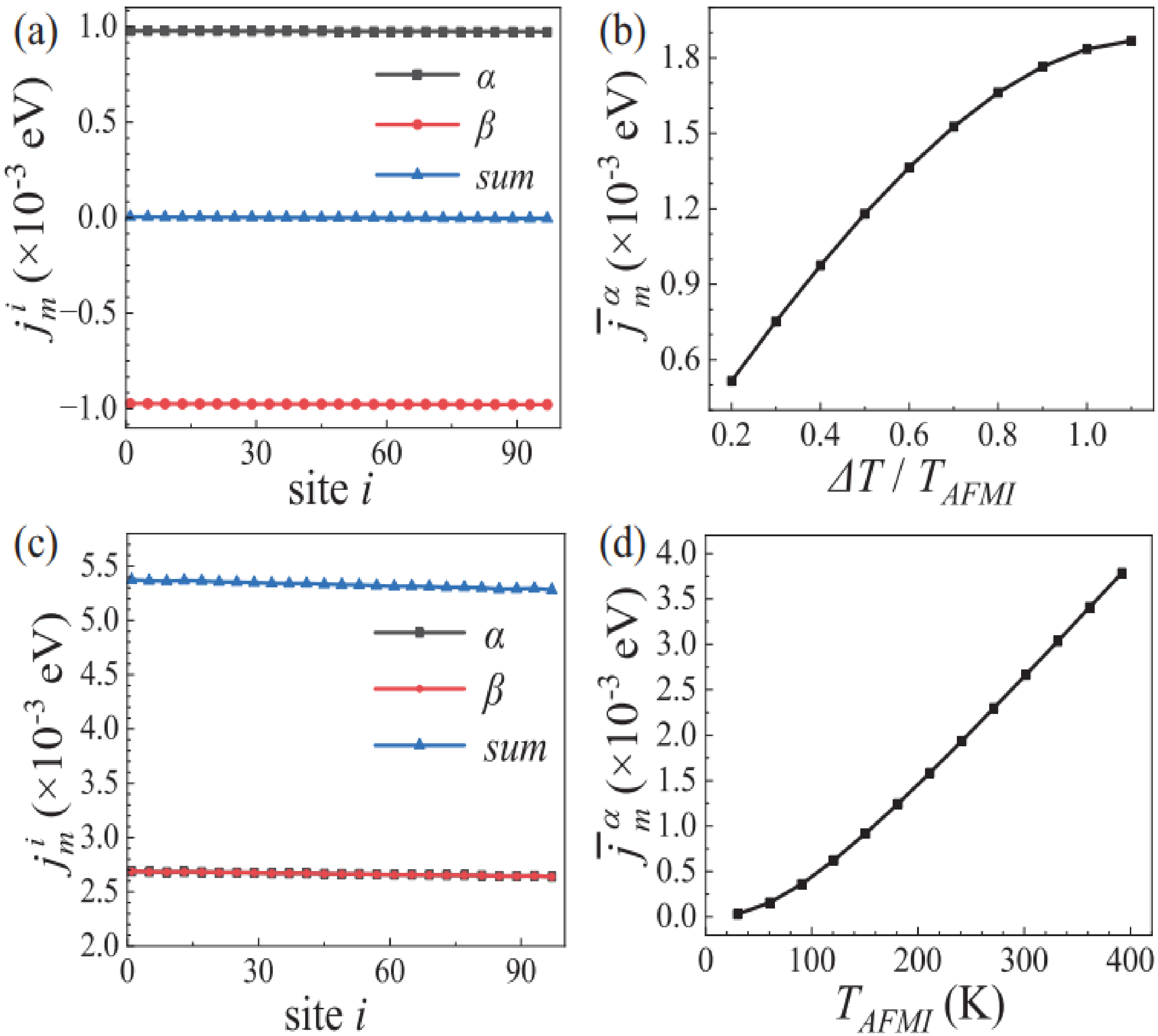
Parameters of (c, d):

$\mu_L = 0.1$,

$\mu_R = 0$

$k_B T_{AFMI} = 0.026$ eV

$T_L = T_{AFMI} = T_R$



Results and discussion - FIMI

Parameters of (a, b):

$\mu_L = \mu_R = 0$

$k_B T_{FIMI} = 0.026$ eV

$T_L = 1.2\ T_{FIMI}$,

$T_R = 0.8\ T_{FIMI}$

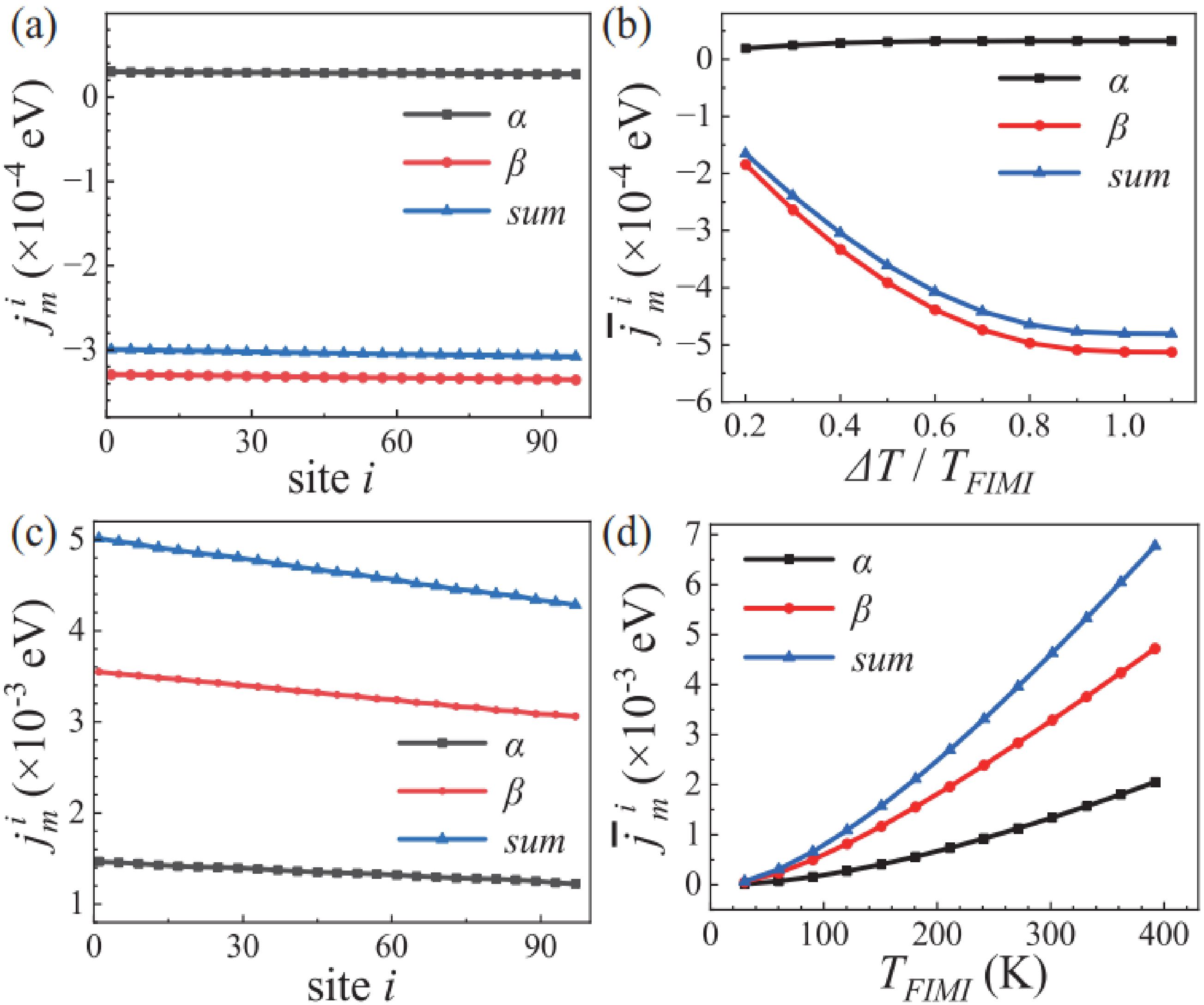
Parameters of (c, d):

$\mu_L = 0.1$,

$\mu_R = 0$

$k_B T_{FIMI} = 0.026$ eV

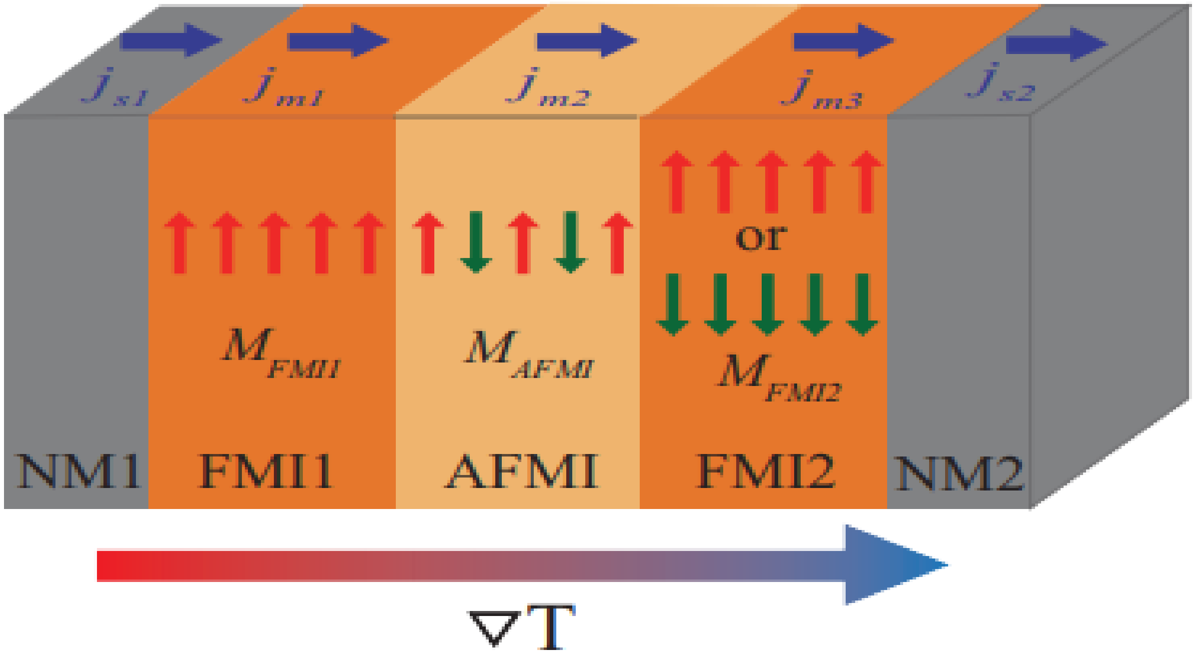
$T_L = T_{FIMI} = T_R$



Results and discussion – Magnon junction

Parameters:

$k_B T_{NM1}$ = 0.026 eV,
T_{FMI1} = 0.9 T_{NM1},
T_{AFMI} = 0.8 T_{NM1},
T_{FMI2} = 0.7 T_{NM1},
T_{NM2} = 0.6 T_{NM1},
S_A = 1, S_B = 1.5



Result:

Parallel state: 6.53 × 10^{-4} eV
Antiparallel state: 4.79 × 10^{-7} eV
Magnon junction ratio MJR = $\frac{J_{m,\uparrow\uparrow}-J_{m,\uparrow\downarrow}}{J_{m,\uparrow\uparrow}+J_{m,\uparrow\downarrow}}$ = 99.85%

Zhang, et al, submitted (2023)

Summary and Outlook

- We proposed a Full quantum theory for magnon transport in two-sublattice magnetic insulators and magnon junctions based on H-P transformation, Fourier transformer and non-equilibrium Green's function.

- Our results can be used to calculate the magnon current induced by temperature gradient and spin chemical potential gradient.

- In the future, our work will be focused on generalizing one-dimensional model, studying the band property of magnon and studying the influence of coupling with photon, phonons on magnon transport.

Thank you very much for your attention!